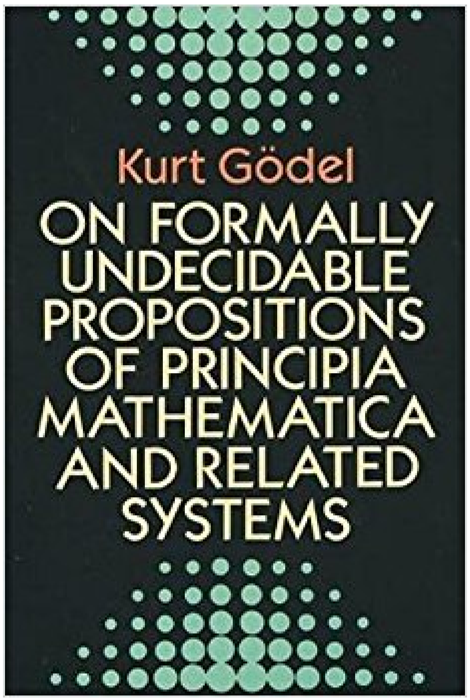
On Formally Undecidable Propositions Of Principia Mathematica And Related Systems



Synopsis

In 1931, a young Austrian mathematician published an epoch-making paper containing one of the most revolutionary ideas in logic since Aristotle. Kurt Giidel maintained, and offered detailed proof, that in any arithmetic system, even in elementary parts of arithmetic, there are propositions which cannot be proved or disproved within the system. It is thus uncertain that the basic axioms of arithmetic will not give rise to contradictions. The repercussions of this discovery are still being felt and debated in 20th-century mathematics.The present volume reprints the first English translation of Giidel's far-reaching work. Not only does it make the argument more intelligible, but the introduction contributed by Professor R. B. Braithwaite (Cambridge University}, an excellent work of scholarship in its own right, illuminates it by paraphrasing the major part of the argument.This Dover edition thus makes widely available a superb edition of a classic work of original thought, one that will be of profound interest to mathematicians, logicians and anyone interested in the history of attempts to establish axioms that would provide a rigorous basis for all mathematics. Translated by B. Meltzer, University of Edinburgh. Preface. Introduction by R. B. Braithwaite.

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Customer Reviews

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This is not a simple read for non-mathematicians, but it is outstanding due to the extended explanatory introduction for others with backgrounds in natural sciences, logical, or philosophical matters.

Requested

good quality.

Difficult to follow.

In my humble opinion one of the most important discoveries of the 20th century. In fact it means that to describe the world with a consistent and complete logical theory, we need an infinit number of hypothesis!

As indicated in two other reviews of mine here, my comprehension of Goedel’s work is opposite to the general one. My marking three stars regardless for this book is motivated by his extensive influence, but also by his fair admission later in life that his thesis could amount to hocus-pocus.Indeed, I see it as one of the prominent mistakes in logical history, and I shall endeavor to explain as best I can. It should suffice to consider his Section 1, an outline of his proposed proof.Although that section is brief, it already foreshadows an oppressingly complex logical symbolism for statements that in my view can be made much clearer using ordinary language. The symbolism, to be sure, is intended to establish a formal language, whose meaning is to be decided separately. This will be seen one of the problems.For now, let me give the principal statement Goedel contended to be true but undecidable (neither provable nor disprovable):"This statement is unprovable."He symbolized it (p.40) as: "~Bew[R(n);n]". Font limitations made me slightly change it; the tilde "~" means "not", "Bew" is a German abbreviation for "provable", and within brackets "R(n)" says "Statement n" and "n" stands for the full statement.Goedel proceeds: "...supposing...~Bew[R(n);n] were provable, it would also be correct; but that means...that...~Bew[R(n);n] would hold good, in contradiction to our initial assumption. If, on the contrary, the negation of ~Bew[R(n);n] were provable, then [its provability] would hold good. ~Bew[R(n);n] would thus be provable [in contradiction to the unprovability it states], which again is impossible." (I corrected some errors within brackets.)So since both ~Bew[R(n);n] and its negation are unprovable, it is undecidable, and Goedel continues (p.41): "...it follows at once that ~Bew[R(n);n] is correct, since...certainly unprovable (because undecidable). So the proposition which is undecidable in the system...turns out to be decided by metamathematical considerations.""Metamathematical", in excusing the contradiction, designates the above formal system void of assigned meaning, whereas the statement discussed is to have meaning. Not quite a lucid argument. Overlooked, furthermore, is a contradiction using the same reasoning as in the preceding.Coupled with the preceding finding that ~Bew[R(n);n] CANNOT be proved unprovable (for if so proved, it would be contradicted), can in contradiction be that it CAN be proved unprovable. For if it were instead provable, it would again be contradicted. The statement in question thus becomes a paradox, rather than true, similar to paradoxes like the "liar", mentioned by Goedel (p.40).He strangely adds to it the footnote: "Every epistemological [paradox] can likewise be used for a similar undecidability proof." The "liar", however, is, like all paradoxes, not a true statement, as required, but one harboring a contradiction. (I deal in my book with, and offer solutions to, paradoxes more fully, including Goedel's resulting one, without naming him.)There occurs, further, another huge blunder in the alleged proof. The undecidability is said to apply to some of mathematics; in the above formula, ~Bew[R(n);n], the "n" refers to a number, with this justification by Goedel (p.38): "For metamathematical purposes it is naturally immaterial what objects are taken as basic signs, and we propose to use natural numbers for them." Adding (p.39): "Metamathematical concepts and propositions thereby become concepts and propositions concerning natural numbers..."How so? In one breath he proposes using natural numbers as immaterial signs, and in the next breath the material concerns natural numbers!The fallaciousness can indeed be made clear by considering our statement, ~Bew[R(n);n], interpreted as "This statement is unprovable." As noted, in ~Bew[R(n);n] the "n", now a number, is to name the whole statement, inside which it is also used in "Statement n..." But whether or not the statement is named by a number, the point is that the name must refer to the intended content of the statement to correspondingly function, not to the usual number possibly represented. Therefore the statement, or anything else similarly used, has nothing to do with numbers, or mathematics generally.

This book is quite short, but it is also very deep. Kurt GÃ¶del was a mathematician back in the 1930s that had an idea. He grew up during a time where it was thought that everything could be explained through mathematics and that mathematics itself would be "complete." However, Kurt GÃ¶del comes up one fine day in 1931 or so and publishes this little paper explaining that there are ideas that can't be expressed in the language of mathematics. Using the language developed by Bertrand Russell and Alfred North Whitehead, Kurt GÃ¶del establishes basic math and then proceeds to tear it down. A tour de force of logic.

Incredibly difficult if not impossibly difficult. You need to be a seasoned logician to figure out even the problem statement. A more enjoyable book is Tarski's book on Logic and leave this on the shelf and pick up a semi-technical version of it. Maybe Einstein his friend probably did not bother reading it and am sure it went through old Albert's hair and pipe too. Also, is really being sneaky charging an extra $2 for this when the back cover clearly says it is $6.95